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AUTHOR Temkin, Sanford
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ABSTRACT

Public sector economic analysis differs from private sector analysis in that the former assumes continuous functions and very small departures from existing conditions while the latter is generally concerned with large discrete either-or decisions. This paper (1) defines an analytic measure that relates effectiveness and cost so that the contributions of system elements to total system effectiveness and total system cost may be appraised, (2) shows how this measure has been applied to health and physical education programs in a school district, and (3) evaluates the measure by highlighting its potential usefulness and limitations. (Reproductions may be poor because of low contrast level of print.) (Author/LLR)

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THE ELASTICITY OF COST-EFFECTIVENESS: IMPLICATIONS FOR PUBLIC SECTOR DECISION-MAKING*

Sanford Temkin

Administering for Change Program
Research for Better Schools, Inc.

Public sector economic analysis differs from private sector analysis in many respects. Some of these differences are apparent while others are quite subtle. Among the more apparent differences between the sectors is the general intangibility and incommensurability of outputs produced by public sector endeavors. This paper considers one of the subtle differences.

The objectives of this paper are 1) to define an analytic measure which relates effectiveness and cost in such a way that the contributions of system elements to total system effectiveness and total system cost may be appraised, 2) to show how this measure has been applied to health and physical education programs in a school district, and 3) to evaluate the measure by highlighting its potential usefulness and limitations.

Economics of the Private Firm

To find optimal solutions (e.g. how much of commodity x to produce) to private sector micro-analysis[1] problems, we employ solutions that have been derived by applying the calculus to continuous functions. The demand curve is an instance of a continuous function which relates, under conditions of pure

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[1]Micro-analysis considers the single firm and its relationships to its markets. Macro-analysis considers the general economy.

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completion, the amount of commodity x that will be demanded at various prices, given consumer preferences, incomes and prices of other commodities.

The economist defines the Elasticity of Demand, E_D, which "measures the rate of proportional decrease in demand for proportional increases in price from the price and demand in question."[2]

This measure may be defined symbolically as

$$E_D = \frac{\frac{dq}{q}}{\frac{dp}{p}} = \frac{p\,dq}{q\,dp}.$$

; dq is a change in the quantity of x demanded
q is the old quantity of x demanded
dp is the change in the price of x
p is the old price of x

To find out what happens to marginal revenue as a result of a price decrease we see

$$\text{marginal revenue} = (p - dp)(q + dq) - pq$$
$$= p\,dq - q\,dp - dpdq.$$

Since we are dealing with continuous curves and the price decrease was assumed to be very small, the term dpdq may be dropped. This indicates that marginal revenue may be analyzed directly from E_D since marginal revenue is equal to p dq - q dp. When E_D is positive, then p dq > q dp. Thus, a positive marginal revenue is another way of saying that total revenue is increased by lowering price.[3]

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[2] R. G. D. Allen. Mathematical Analysis for Economists. (London: MacMillan & Co. Ltd., 1962), p. 255.

[3] For a full development of these concepts as well as where they lead see, for example, Sidney Weintraub. Intermediate Price Theory. (Philadelphia: Chilton Books, 1964).

The Elasticity of Cost-Effectiveness

This brings us to the central theme of this paper. The very essence of private sector micro-analysis hinges on continuous functions and very small incremental departures from existing conditions. Public sector micro-analysis, however, is concerned with decisions involving large incremental departures from existing conditions. In education we encounter, for instance, decisions about whether or not to fund project x, or whether or not to add a reading specialist.

One way to evaluate alternatives is to look at the system "with and without" the particular element being evaluated. This method approximates the method of marginal analysis. Implicit in the "with and without" notion is the separability of the effectiveness and cost contributions of the elements in the system.

The Elasticity of Cost-Effectiveness for project i is defined as

$$(E/C)_i = \frac{\Delta E/(E - \Delta E)}{\Delta C/(C - \Delta C)} ; \quad \Delta E < E , \quad \Delta C < C,$$

and where ΔE is the relative effectiveness score[4] for project i,

E is the combined relative effectiveness score for all M projects in the system, $E = \sum_{i=1}^{M} E_i$,

ΔC is the cost of project i,

C is the total cost for all M projects, $C = \sum_{i=1}^{M} C_i$.

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[4]It should be added that the concept of relative effectiveness includes a performance index variable ($0 \leq P_i \leq 1.0$) and a relative importance assignment ($0 \leq V_i \leq V_{MAX}$). Consequently, $E = f(P, V)$. For a full development of the concept of relative effectiveness as a function of value and performance see, Sanford Temkin. "A Comprehensive Theory of Cost-Effectiveness," Philadelphia: Research for Better Schools, April, 1970.

We can rearrange terms in order to examine some properties of (E/C)i. That is,

$$(E/C)_i = \frac{\Delta E(C - \Delta C)}{\Delta C(E - \Delta E)} = \frac{C\Delta E - \Delta C\Delta E}{E\Delta C - \Delta C\Delta E} .$$

The fundamental observation is that the public sector decision-maker cannot take the term $\Delta C\Delta E$ as inconsequential. Primarily this is due to the very essence of public sector decision-making in which the decision-maker is often faced with large "either-or" decisions, as opposed to incremental decisions such as should we produce 5,000 or 5,001 units.

Now we can rearrange terms to arrive at a meaningul expression of the cost and effectiveness variables. That is,

$$(E/C)_i = \frac{C\Delta E - \Delta C\Delta E}{E\Delta C - \Delta C\Delta E} = \frac{\Delta E}{\Delta C} \cdot \frac{C - \Delta C}{E - \Delta E}$$

$$(E/C)_i = \frac{\frac{\Delta E}{\Delta C}}{\frac{E - \Delta E}{C - \Delta C}} \begin{matrix} > \\ = \\ < \end{matrix} 1, \quad \begin{matrix} C > \Delta C & > 0 \\ E > \Delta E & \geq 0 \end{matrix} .$$

The numerator of (E/C)i is the ratio of the effectiveness to the cost of the project being evaluated and the denominator is the ratio of the rest of the system's effectiveness to its cost. When the ratio of the ratios is unity, the project being evaluated does not alter the average effectiveness-cost ratio for the system. A ratio less than 1.0 reflects the evaluation of a project producing at a lower rate per dollar than the average of the other projects of the system. A ratio greater than 1.0 indicates that a project is being evaluated which has a higher rate of productivity per unit of input than the average of the remaining system projects.

A Perspective for (E/C)i.

I believe, and I would hope that you would share this belief, that after-the-fact evaluation of cost-effectiveness performance is only helpful to tell the decision-maker where he is. What is more important, it seems to me, is what he should do next. Consequently, cost and effectiveness information for decision-making purposes should be made up of future oriented estimates of cost and effectiveness for alternatives under consideration. (To the extent that projects may be viewed incrementally, the decision-maker will be able to consider trade-offs among alternative decisions).

A Note on Suboptimization and Strategies

When an increase in expenditure for a project within a program gives the program an increase in its elasticity-figure but gives the system of programs a decrease in the ratio of total effectiveness to total cost, we may be considering a suboptimal allocation.[5] A suboptimal strategy gives resources to each program, which then makes the best allocation it can. An optimal strategy examines projects and allocates resources at the level of the top decision-maker in the organization.

The appendix to this paper empirically examines implications of optimal and suboptimal strategies by effectiveness and cost criteria.

Conclusions

School systems could benefit from better information about the effectiveness

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[5]It is conceivable that the project under consideration may represent to the organization the best candidate for increased expenditure and yet still result in a decrease in the overall ratio of effectiveness to cost. Under these conditions the organization is operating past peak efficiency -- that is, at the point of diminishing marginal and average effectiveness/cost.

and costs of their operations. Projects and specific changes in schools lend themselves to better management and evaluation procedures. Ongoing programs are more difficult to evaluate.

The Elasticity of Cost-Effectiveness could be a useful measure if effectiveness measures can be defined, accounting systems can be modified, and genuine alternatives to the ongoing program can be explored in terms of effectiveness and costs.

APPENDIX

This appendix presents health and physical education program data derived during 1967-68.

Table 1 shows effectiveness and cost data for the "present year" (1967-68) and Table 2 shows results conditional upon each of three budgeting strategies being applied to a sum of money to be allocated to the programs.

The optimal allocation strategy allocates funds to programs by finding the increment promising the best estimated effectiveness return per dollar from the remaining available alternatives at any given point in the allocation sequence.

The suboptimal allocation strategy allocates a previously determined sum to the best estimated effectiveness per dollar in each program. This guarantees that every program will have a budgeted amount greater than its 1967-68 expenditure level.

The make-everyone-happy strategy allocates as much funds as are requested by each program, irrespective of estimated effectiveness and costs.

Allocations in Table 2 are more effectively evaluated by examing three criteria:

1. The overall system relative effectiveness, $\frac{\Sigma VP}{\Sigma V} = \frac{\Sigma E}{\Sigma V}$,
2. The overall system effectiveness-cost ratio, $\frac{\Sigma E}{\Sigma C}$,
3. The Elasticity of Cost-Effectiveness for each program.

TABLE 1

COST AND EFFECTIVENESS INFORMATION FOR THE PRESENT YEAR

Program*	Present Year' Cost ($)	Present Year Effectiveness	$\frac{\text{Effectiveness}}{\text{Cost}} \times \$1000$	Elasticity of Cost-Effectiveness
P_m	C_m	E_m	E_m/C_m	(E/C)m
P_1	2,961	50.9	17.2	1.98
P_2	3,385	72.9	21.5	2.59
P_3	913	30.8	33.7	3.87
P_4	3,595	156.6	43.6	6.55
P_5	180	5.0	**	3.06
P_6	3,000	18.1	6.0	0.65
P_7	27,259	110.2	4.0	0.28
P_8	11,825	41.6	3.5	0.33
All programs	$53,118	486.1	9.3***	**

*The term "Program" is preferred rather than the term "Project" because program implies an ongoing activity. The costs and effectiveness of a program may be analyzed after fiscal period or for purposes of future oriented decisions but project costs and effectiveness during a given year may describe no more than some elements on the path to an end product. Projects, to be adequately analyzed, must be received in terms of the full past and future expenditure intentions as well as the gains that are expected to be accrued by target populations over time. Program analysis has different parameters.

**not a reasonable calculation as defined in this paper.

***based on $\Sigma E_m/\Sigma C_m$

If we use the cost and effectiveness information for the present year as a starting point, then it is possible to evaluate the implications for proposed charges from the status quo. Two sets of information are very useful. The Elasticity of Cost-Effectiveness figure for a given program compares this average to the averages of all other programs. The overall effectiveness/cost figure for all programs combined tells about average efficiency for the system.

Table 2 describes the results of three expenditure strategies for a budget increase.

TABLE 2

THE EXPECTED RESULTS* OF THREE EXPENDITURE STRATEGIES

Program P_m	Present Year Elasticity	Optimal Allocation Elasticity	Suboptimal Allocation Elasticity	Allocate funds to meet all requests Elasticity
P_1	1.98	1.51	1.64	1.87
P_2	2.59	2.70	2.26	1.76
P_3	3.87	2.75	2.99	2.51
P_4	6.55	5.29	5.75	6.48
P_5	3.06	3.19	3.46	3.91
P_6	0.65	0.62	0.68	0.69
P_7	0.28	0.32	0.38	0.37
P_8	0.33	0.35	0.29	0.32
Relative Effectiveness	0.81	0.87	0.88	0.90
$\Sigma E/\Sigma C$ x \$1000	9.3	8.8	8.1	7.2

*Raw data are not shown as in Table 1. Raw data were taken from cost-effectiveness curves for each program.

Conclusions

If we allocate optimally, average efficiency falls indicating that marginal efficiency must be falling. The price that one pays for suboptimization and finally honoring all requests for funds can also be observed by comparing the overall effectiveness to cost ratio that would be expected if each of the respective strategies were implemented.

Relative effectiveness, of course, rises as more money is expended since it has been defined as an index.

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